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BOOK REVIEWS.

EDITED BY W. H. BUSSEY, University of Minnesota.

A New Analysis of Plane Geometry, Finite and Differential. By A. W. H. THOMPSON, Sometime Scholar of Trinity College. Cambridge University Press, 1914. xvi + 120 pages.

From the preface of this interesting book we quote: "There are two main ways of considering Geometry. One is by sight or figure. The other is by a symbolic representation of the figure. The former method I call the Visual method; the latter the Symbolic method. . . . The method of the text belongs to the Symbolic method. In some cases, however, the process is easily visualized and coincides with the treatment of Visual Geometry. . . . I claim the method [of this text] as original. There are some theorems which are original, and most of the general results in the examples are new. The new treatment of the trigonometric functions is original and necessary for the purpose." The author remarks further upon the similarity of the symbols in this book and in the method of Grassman although he was not aware of Grassman's method when he discovered the method used here.

The book consists of two parts, Finite geometry and Differential geometry. The part on finite geometry contains an introduction and three chapters on the reduction of the three ways in which the elements of the geometry, points and lines, may be expressed. The nine chapters in the differential geometry are concerned with the differentiation of these expressions, the reduction of measures containing functional elements, generalized displacement of a point, the reduction of complex functional elements, the successive differentiation of measures, and integration of measures.

In the notation of this book the small Italic letters $a, b, c, \cdots, x, y, z$ denote points and the small Greek letters $\alpha, \beta, \gamma, \cdots, \omega$ denote lines. The determinate of two points is the line incident in the points and is denoted by $\overline{xy}$, while the determinate of two lines $\overline{\alpha\beta}$ is their point of intersection. The measure of two points (xy) is the distance between them; the measure of two lines $(\alpha\beta)$ is the angle between them; the measure of a point and a line $(a\gamma)$ is the perpendicular distance from a to γ. Then $(\overline{ab}\ \overline{\gamma\delta})$ would be the perpendicular from the point of intersection of the two lines to a line determined by the points a and b. The measure (xy) is distinct from (yx) and the measure $(\alpha\beta)$ is distinct from $(\beta\alpha)$ so that lines and angles have a "sense" of direction. The general vectorial point may be expressed as $a_{\rho'\theta'\phi'}$ where ρ' is a vector having a direction ρ while the distance of the point from a is denoted by the magnitude ρ. The line $a_{\rho'\theta'\omega}$ is incident in the point $a_{\rho'\theta'}$ and has the direction ω. Furthermore lines and points may be expressed in a third way in equational elements as the point $A_r(\xi a_r) = 0$. The measure (xyz) is shown to be equal to twice the area of the triangle of which the points x, y, and z are the vertices. The chief part of the first three chapters is the reduction of measures of several elements to those of two elements. The definition of this method of reduction is not expressly given but when one supplies this omission, the remainder of the process follows easily. Thus the definition $(xyz) = (xy)(\overline{xy}z)$ proves the theorem just mentioned. The definition is more apparent in $(\alpha\beta cd) = \sin(\alpha\beta)(\overline{\overline{\alpha\beta}cd}) = \sin(\alpha\beta)(\overline{\alpha\beta}c)(\overline{\overline{\alpha\beta}c}d)$ which reduces to $(c\alpha)(d\beta) - (c\beta)(d\alpha)$. All the formulas derived for the first method of representation of points and lines are applicable to the vectorial and equational representations. The following problem given as an exercise may be proved by two very simple steps: Let $\left.\frac{abc}{\alpha\beta\gamma}\right\}$ be a triangle and λ any line: p, q, r are the feet of the perpendiculars from a, b, c on λ: show that the perpendiculars from p, q, r on α, β, γ, are concurrent.

In the differential geometry, if x is any point, and x' a consecutive point, then (xx') is denoted by δx and $\overline{xx'}$ by τx, where τx is the line tangent to a curve at a point x upon it. νx is the normal line to a curve at a point x upon it. If ξ is any line and ξ' is a consecutive line, then $(\xi\xi')$ is denoted by $\partial\xi$ and $\overline{\xi\xi'}$ by $p\xi$. From these simple conditions are found the derivatives of the measures which were deduced in the first three chapters.

At the end of the book are given 108 formulas which are derived in the body of the work. At the end of each chapter is a collection of examples and following Chapter XII is a list of 106 miscellaneous examples. The method of the book may be shown best by a simple problem. "Show that the product of the perpendiculars from the foci of an ellipse to any tangent is constant."

Let $|(xs)| + |(xs')| = k$ be the equation of the ellipse where s and s' are the foci and x is a variable point upon the curve.

Differentiating,

$$\frac{(xs)^2}{(\nu xs)^2} = \frac{(xs')^2}{(\nu xs')^2},$$

then

$$\frac{(\nu xs)}{(\tau xs)} + \frac{(\nu xs')}{(\tau xs')} = 0, \quad \text{that is,} \quad \frac{(\nu\xi s)}{(\xi s)} + \frac{(\nu\xi s')}{(\xi s')} = 0,$$

by putting $\tau x = \xi$.

Integrating,

$$\log(\xi s) + \log(\xi s') = \text{constant},$$

$$\therefore \quad (\xi s)(\xi s') = \text{constant}.$$

While this is entirely symbolic, the reader may obtain some idea of the beauty of this method by drawing the figure and seeking a visual interpretation of each step. Of course we have given no clue to the methods of differentiation or integration. We see in this book several theorems on the cross-ratio of a pair of points a, b and a pair of lines γ, δ, whose definition is $\frac{(a\gamma)/(b\gamma)}{(a\delta)/(b\delta)}$ and is written $\{ab, \gamma\delta\}$. Another interesting problem is, show that $(xa\beta c\delta ex) = 0$ is the equation of a conic and deduce Pascal's theorem.

This geometry will never replace the older geometries, but with further development many new theorems may be discovered. Any one interested in modern geometry will read the work with pleasure; and detecting the few slight errors will add a little zest.

F. A. Foraker.

University of Pittsburgh.

PROBLEMS AND SOLUTIONS.

Edited by B. F. Finkel and R. P. Baker.

PROBLEMS FOR SOLUTION.

ALGEBRA.

443 Proposed by A. M. KENYON, Purdue University.

If p_r denote the sum of all the different r-factor products that can be formed from the first n natural numbers ($p_r = 0$ for $r > n$), and if

$$D_s = \begin{vmatrix} p_1 & 1 & 0 & \cdots & 0 \\ 2p_2 & p_1 & 1 & \cdots & 0 \\ 3p_3 & p_2 & p_1 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ sp_s & p_{s-1} & p_{s-2} & \cdots & p_1 \end{vmatrix}$$

show that

$$\sum_{i=0}^{k} (-1)^i c_i \binom{k}{i} D_{2k-i} = 0, \qquad k, n = 1, 2, 3, \cdots,$$

where $c_i = \dfrac{2k+1-i}{1+i}$ when i is even and $2n+1$ when i is odd; and $\binom{k}{i}$ is the coefficient of x^i in $(1+x)^k$.

444. Proposed by J. E. ROWE, Pennsylvania State College.

Prove that the determinant

$$\begin{vmatrix} \cot A & \cot B & \cot C \\ 1 & 1 & 1 \\ \cos^2 A & \cos^2 B & \cos^2 C \end{vmatrix} = 0,$$

where A, B, and C are the angles of a plane triangle.

GEOMETRY.

474. Proposed by LAENAS G. WELD, Pullman, Illinois.

Upon a fixed and constant base stands a system of co-planar triangles, for each of which the radius of the inscribed circle is to that of the circumscribed circle as 1 : 2. What is the locus of the vertices opposite to the given fixed base?

475. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Given two circles and a straight line, to draw a circle tangent to the line and coaxial with the two given circles.